기 계 진 동 학 (제4판) 연습문제 풀이 제3장~끝 (2023년 과제)

[3.1절]

3.8 알루미늄의 영률 $E = 71\times10^9$ Pa (=N/m^2)

$m = 2{,}000$ kg, $L = 3$ m, $\zeta = 0.02$, $I = 1.0$ m^4, $F = 2{,}000$ N, $\Delta t = 0.1$ s

강성 $k = \dfrac{3EI}{L^3} = \dfrac{3\,(71\times10^9\ \text{N/m}^2)(1.0\ \text{m}^4)}{(3\ \text{m})^3} = 7.89\times10^9$ (N/m)

$\omega_n = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{7.89\times10^9\ \text{N/m}}{2{,}000\ \text{kg}}} = 1{,}986$ rad/s,

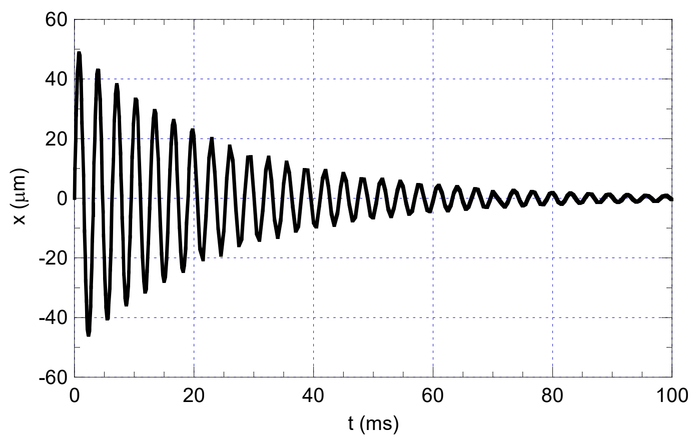
$\zeta\,\omega_n = (0.02)(1{,}986\ \text{rad/s}) = 39.7$ rad/s

$\omega_d = \omega_n\sqrt{1-\zeta^2} = (1{,}986\ \text{rad/s})\sqrt{1-0.02^2} = 1{,}986$ rad/s

$x(t) = \dfrac{(F\,\Delta t)}{m\,\omega_d}\,e^{-\zeta\omega_n t}\,\sin\omega_d t = \dfrac{(2{,}000\ \text{N})(0.10\ \text{s})}{(2{,}000\ \text{kg})(1{,}986\ \text{rad/s})}\,e^{-39.7t}\,\sin(1{,}986\ t)$

$= 50.4\times10^{-6}\,e^{-39.7t}\,\sin(1{,}986\ t)$ m $= 50.4\,e^{-39.7t}\,\sin(1{,}986\ t)$ μm

Plot



[3.2절]

3.28 사각하중 응답 (유한 시간 계단함수 하중에 대한 응답)

F_0 = 30 N, $t_1 = \dfrac{\pi}{\omega_n}$, k = 1,000 N/m, ζ = 0.1, ω_n = 3.16 rad/s

$\zeta\omega_n$ = (0.1) (3.16 rad/s) = 0.316 rad/s, $\omega_d = \sqrt{1-0.1^2}$ (3.16 rad/s) = 3.144 rad/s

$$t_1 = \frac{\pi\,\text{rad}}{3.16\,\text{rad/s}} = 0.994\ \text{s}$$

$0 \le t \le 0.994$ s 일 때,

$$x(t) = \frac{F_0}{k}\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \phi)\right]$$

$$\phi = \tan^{-1}\frac{\zeta}{\sqrt{1-\zeta^2}} = \tan^{-1}\frac{0.1}{\sqrt{1-0.1^2}} = 0.1002\ \text{rad}$$

$$\frac{F_0}{k} = \frac{30\ \text{N}}{1{,}000\ \text{N/m}} = 0.03\ \text{m},\quad \frac{1}{\sqrt{1-0.1^2}} = 1.005$$

$\Rightarrow$ $x(t)$ = 0.0300 [1 - 1.005 $e^{-0.316t}$ cos(3.14 t - 0.1002)] (m)

$t > 0.994$ s 일 때,

[방법1] 예제 3.2.2 교재의 풀이 방법

$$x(t) = \frac{F_0}{k}\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t - \phi)\right] - \frac{F_0}{k}\left\{1 - \frac{e^{-\zeta\omega_n(t-t_1)}}{\sqrt{1-\zeta^2}}\cos[\omega_d(t-t_1)-\phi]\right\}$$

$$= \frac{F_0}{k\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\left\{e^{\zeta\omega_n t_1}\cos[\omega_d(t-t_1)-\phi] - \cos(\omega_d t - \phi)\right\}$$

$$\frac{F_0}{k\sqrt{1-\zeta^2}} = \frac{(30\ \text{N})(1.005)}{1000\ \text{N/m}} = 0.03015\ \text{m}$$

$\zeta\omega_n t_1$ = (0.316 rad/s) (0.994 s) = 0.314 rad

$e^{\zeta\omega_n t_1} = e^{0.314}$ = 1.369

$\omega_d(t-t_1) - \phi$ = (3.144 rad/s) [t - (0.994 s)] - (0.1002 rad)

= (3.144 rad/s) t - (3.225 rad)

$\Rightarrow$ $x(t)$ = 0.0302 $e^{-0.316t}$ [1.369 cos(3.14 t - 3.23)

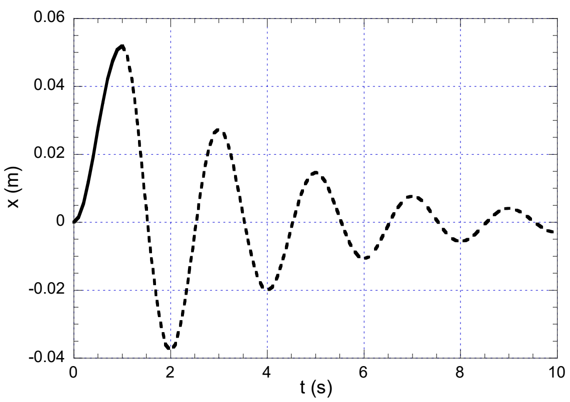
- cos(3.14 t - 0.1002)] m

[방법2] 예제 3.2.2 노트의 풀이 방법

$F(t-\tau) = F_0$ $\quad 0 < t-\tau < t_1 \quad \Rightarrow \quad -t < -\tau < t_1 - t \quad \Rightarrow \quad t > \tau > t - t_1$

$$x(t) = \frac{1}{m\omega_d}\int_{t-t_1}^{t} F_0\, e^{-\zeta\omega_n\tau}\sin\omega_d\tau\, d\tau = \cdots$$

또는 $x(t) = \dfrac{1}{m\omega_d}\displaystyle\int_0^{t_1} F_0\, e^{-\zeta\omega_n(t-\tau)}\sin\omega_d(t-\tau)\, d\tau = \cdots$



[3.3절]

3.35 삼각파(예제 3.3.1) 가진에 대한 1자유도 감쇠계의 **정상상태** 응답 (그래프 plot 생략)

최대 힘 1 N, m = 80 kg, ζ = 0.1, k = 1000 N/m, $T = 2\pi$ s

$$\omega_n = \sqrt{\frac{1000\ \text{N/m}}{80\ \text{kg}}} = \frac{5}{\sqrt{2}}\ \text{rad/s} = 3.536\ \text{rad/s},\quad \omega_T = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1\ \text{rad/s}$$

$$F(t) = \sum_{n=1,3,\ldots}^{\infty} \frac{-8}{n^2\pi^2}\cos\frac{2n\pi}{T}t = \sum_{n=1,3,\ldots}^{\infty} \frac{-8}{n^2\pi^2}\cos nt\ ,\qquad a_n = \frac{-8}{n^2\pi^2}$$

$$x_n^c(t) = X_n\ \cos(n\omega_T t - \theta_n)$$

$$X_n = \frac{a_n/m}{\sqrt{[\omega_n^2-(n\omega_T)^2]^2+(2\zeta\omega_n n\omega_T)^2}}$$

$$= \frac{-8}{(80)n^2\pi^2}\frac{1}{\sqrt{[\frac{25}{2}-n^2]^2+[2(0.1)\frac{5}{\sqrt{2}}n]^2}} = \frac{-0.01013}{n^2\sqrt{(\frac{625}{4}-25n^2+n^4)+0.5n^2}}$$

$$\theta_n = \tan^{-1}\frac{2\zeta\omega_n n\omega_T}{\omega_n^2-(n\omega_T)^2} = \tan^{-1}\frac{2(0.1)\frac{5}{\sqrt{2}}n}{\frac{25}{2}-n^2} = \tan^{-1}\frac{0.707\,n}{12.5-n^2}\qquad (\ 0 < \theta_n < \pi\ )$$

$$x_p(t) = \sum_{n=1,3,\ldots}^{\infty} x_n^c(t) = \sum_{n=1,3,\ldots}^{\infty} X_n\ \cos(n\omega_T t - \theta_n)$$

$$= \sum_{n=1,3,\ldots}^{\infty} \frac{-0.01013}{n^2\sqrt{n^4-24.5n^2+156.3}}\cos\left[nt - \tan^{-1}\frac{0.707\,n}{12.50-n^2}\right]\quad \text{m}$$

[3.4절]

3.43 $\ddot{x}(t) + x(t) = \sin 2t,\qquad x_0 = 0,\quad v_0 = 1$ m/s

$$[s^2\ X(s) - s\ x_0 - v_0\ ] + X(s) = \frac{2}{s^2+2^2}$$

$$(s^2+1)\ X(s) = \frac{2}{s^2+2^2} + 1 \quad\Rightarrow\quad X(s) = \frac{2}{(s^2+1)(s^2+2^2)} + \frac{1}{s^2+1}$$

$$X(s) = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+2^2}$$

$$X(s)\ (s^2+1)\big|_{s=i} \quad\Rightarrow\quad A(i)+B = \frac{2}{(s^2+2^2)}\bigg|_{s=i} + 1 = \frac{5}{3} \quad\Rightarrow\quad A = 0,\quad B = \frac{5}{3}$$

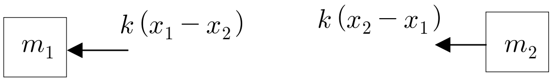
$$X(s)\ (s^2+4)\big|_{s=2i} \quad\Rightarrow\quad C(2i)+D = \frac{2}{(s^2+1)}\bigg|_{s=2i} = -\frac{2}{3} \quad\Rightarrow\quad C = 0,\quad D = -\frac{2}{3}$$

$$X(s) = \frac{5}{3}\frac{1}{s^2+1} - \frac{1}{3}\frac{2}{s^2+2^2}$$

$$x(t) = L^{-1}\left[\frac{5}{3}\frac{1}{s^2+1} - \frac{1}{3}\frac{2}{s^2+2^2}\right] = \frac{5}{3}\ \sin t - \frac{1}{3}\ \sin 2t\quad (\text{m})$$

[4.1절]

4.13 자유물체도



운동방정식

$$m_1 \ddot{x}_1 = -\ k(x_1 - x_2) \qquad \Rightarrow \qquad m_1 \ddot{x}_1 + k x_1 - k x_2 = 0$$

$$m_2 \ddot{x}_2 = -\ k(x_2 - x_1) \qquad \Rightarrow \qquad m_2 \ddot{x}_2 - k x_1 + k x_2 = 0$$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{Bmatrix} + \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

고유진동수

$$\det\ (-\omega_n^2\ M + K) = \det \begin{bmatrix} -\omega_n^2 m_1 + k & -k \\ -k & -\omega_n^2 m_2 + k \end{bmatrix}$$

$$= (-\omega_n^2 m_1 + k)(-\omega_n^2 m_2 + k) - (-k)(-k)$$

$$= m_1 m_2\ \omega_n^4 - (m_1 + m_2)\ k\ \omega_n^2 + k^2 - k^2$$

$$= m_1 m_2\ \omega_n^4 - (m_1 + m_2)\ k\ \omega_n^2$$

$$= [m_1 m_2\ \omega_n^2 - (m_1 + m_2)\ k]\ \omega_n^2 = 0$$

$$\Rightarrow \quad \omega_1 = 0, \quad \omega_2^2 = \frac{(m_1 + m_2)k}{m_1 m_2}$$

$m_1 = m_2 = 2{,}100$ kg, $\quad k = 270{,}000$ N/m

$$\Rightarrow \quad \omega_2^2 = \frac{(2{,}100 + 2{,}100\ \mathrm{kg})(270{,}000\ \mathrm{N/m})}{(2{,}100\ \mathrm{kg})(2{,}100\ \mathrm{kg})} = 257.1\ (\mathrm{rad/s})^2$$

$$\Rightarrow \quad \omega_2 = 16.04\ \mathrm{rad/s}$$

모드 형상

$$\begin{bmatrix} -\omega_n^2 m_1 + k & -k \\ -k & -\omega_n^2 m_2 + k \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \quad (-\omega_n^2 m_1 + k)\ u_1 - k u_2 = 0$$

$$\Rightarrow \quad u_2 = \frac{-\omega_n^2 m_1 + k}{k} u_1$$

$\omega_n = \omega_1 = 0$ 일 때,

$$u_2 = u_1 \qquad \Rightarrow \qquad \mathbf{u}_1 = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix}$$

$\omega_n = \omega_2 = 16.04$ rad/s 일 때,

$$u_2 = \frac{-(257.1\ \mathrm{rad^2/s^2})\ (2{,}100\ \mathrm{kg}) + (270{,}000\ \mathrm{N/m})}{270{,}000\ \mathrm{N/m}} u_1 = (-1)u_1 \qquad \Rightarrow \qquad \mathbf{u}_2 = \begin{Bmatrix} 1 \\ -1 \end{Bmatrix}$$

[1.6절]

1.94 그래프로부터 $T = 4\ \text{s},\ x_1 = 0.75\ \text{cm},\ x_2 = 0.21\ \text{cm}$

$\omega_d = \dfrac{2\pi\ \text{rad}}{4\ \text{s}} = \dfrac{\pi}{2}\ \text{rad/s} = 1.571\ \text{rad/s}$ ⇒ $\omega_d = 1.571\ \text{rad/s}$

$\delta = \ln\dfrac{x_1}{x_2} = \ln\dfrac{0.75}{0.21} = \ln 3.571 = 1.273$

$\zeta = \dfrac{\delta}{\sqrt{4\pi^2+\delta^2}} = \dfrac{(1.273)}{\sqrt{4\pi^2+(1.273)^2}} = 0.19856$ ⇒ $\zeta = 0.1986$

$\omega_n = \dfrac{\omega_d}{\sqrt{1-\zeta^2}} = \dfrac{1.571\ \text{rad/s}}{\sqrt{1-0.19856^2}} = 1.6027\ \text{rad/s}$ ⇒ $\omega_n = 1.603\ \text{rad/s}$