기 계 진 동 학 (제4판) 연습문제 풀이 제2장 (2023년 과제)

[2.1절]

2.18 $\ddot{x}(t) + 4x(t) = 10\cos 2t$, $x(0) = 0$, $\dot{x}(0) = 0$

$\omega_n^2 = 4 \quad \Rightarrow \quad \omega_n = 2$ rad/s, $\omega = 2$ rad/s, $f_0 = 10$ N/kg

$\omega = \omega_n$

$$x(t) = A_1 \sin\omega_n t + A_2 \cos\omega_n t + \frac{f_0}{2\omega}\, t \sin\omega t$$

$$\dot{x}(t) = \omega_n A_1 \cos\omega_n t - \omega_n A_2 \sin\omega_n t + \frac{f_0}{2\omega} \sin\omega t + \frac{f_0}{2}\, t \cos\omega t$$

$$x(0) = A_2 = 0$$

$$\dot{x}(0) = \omega_n A_1 = 0 \quad \Rightarrow \quad A_1 = 0$$

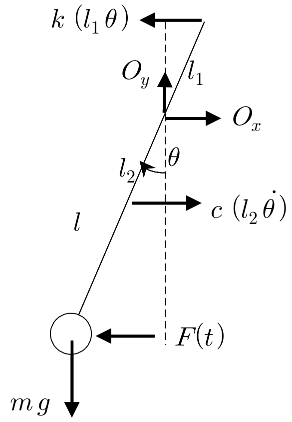
$$\frac{f_0}{2\omega} = \frac{10\ \mathrm{m/s^2}}{2\,(2\ \mathrm{rad/s})} = 2.50\ \mathrm{m/s}$$

$$x(t) = \frac{f_0}{2\omega}\, t \sin\omega t = (2.50\ \mathrm{m/s})\, t \sin 2t$$

[2.2절]

2.27 $\Sigma M_o = J\ddot{\theta}$

F. B. D.



$$-k\,(l_1\theta)\,l_1 - c\,(l_2\dot{\theta})\,l_2 - m\,g\,l\,\theta + F(t)\,l = (m\,l^2)\,\ddot{\theta}$$

$$\Rightarrow \quad (m\,l^2)\,\ddot{\theta} + (c\,l_2^2)\,\dot{\theta} + (k\,l_1^2 + m\,g\,l)\,\theta = l\,F(t)$$

$$J = m\,l^2 ,\quad c_t = c\,l_2^2 ,\quad k_t = k\,l_1^2 + m\,g\,l$$

$$\omega_n = \sqrt{\frac{k_t}{J}} = \sqrt{\frac{k\,l_1^2 + m\,g\,l}{m\,l^2}}$$

$$\zeta = \frac{c_t}{2\sqrt{J k_t}} = \frac{c\,l_2^2}{2\sqrt{(m\,l^2)\,(k\,l_1^2 + m\,g\,l)}}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

$$= \sqrt{\frac{k\,l_1^2 + m\,g\,l}{m\,l^2}}\,\sqrt{1 - \frac{(c\,l_2^2)^2}{4\,(m\,l^2)\,(k\,l_1^2 + m\,g\,l)}}$$

$$= \sqrt{\frac{kl_1^2 + mgl}{ml^2} - \frac{(c\,l_2^2)^2}{4\,(m\,l^2)^2}}$$

resonance at $\omega = \omega_p$

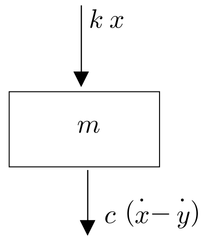
$$\omega_p = \omega_n \sqrt{1 - 2\zeta^2}$$

$$= \sqrt{\frac{k\,l_1^2 + m\,g\,l}{m\,l^2}}\,\sqrt{1 - \frac{2\,(c\,l_2^2)^2}{4\,(m\,l^2)\,(k\,l_1^2 + m\,g\,l)}}$$

$$= \sqrt{\frac{kl_1^2 + mgl}{ml^2} - \frac{(c\,l_2^2)^2}{2\,(m\,l^2)^2}}$$

[2.4절]

2.50 응용된 바닥가진 문제 F. B. D.



$$y(t) = Y \sin\omega_b t$$

$$-kx - c(\dot{x}-\dot{y}) = m\ddot{x}$$

$$\Rightarrow \quad m\ddot{x} + c\dot{x} + kx = c\dot{y}$$

$$= c\,\omega_b\, Y \cos\omega_b t$$

$$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = 2\zeta\omega_n\omega_b Y \cos\omega_b t$$

$$x_p(t) = \frac{2\zeta\omega_n\omega_b Y}{\sqrt{(\omega_n^2-\omega_b^2)^2+(2\zeta\omega_n\omega_b)^2}}\cos(\omega_b t - \theta)$$

$$F_{tr}(t) = k x_p = F_T \cos(\omega_b t - \theta)$$

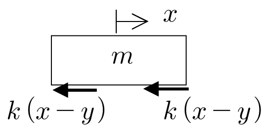
$$F_T = \frac{k\,2\zeta\,\omega_n\,\omega_b\,Y}{\sqrt{(\omega_n^2-\omega_b^2)^2+(2\zeta\omega_n\omega_b)^2}} = \frac{\frac{ck}{m}\omega_b Y}{\sqrt{(\frac{k}{m}-\omega_b^2)^2+(\frac{c}{m}\omega_b)^2}}$$: 최종 답 (문제의 기호로 표현)

(참고 사항 :)

$$= \frac{(kY)\,2\zeta\frac{\omega_b}{\omega_n}}{\sqrt{\left(1-\frac{\omega_b^2}{\omega_n^2}\right)^2+\left(2\zeta\frac{\omega_b}{\omega_n}\right)^2}} = kY\frac{2\zeta r}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

2.60 $Y = 0.1$ m, $\omega_b = 7.5$ rad/s, $m = 10^5$ kg, $k = 3.519\times10^6$ N/m, $\zeta = 0$

F. B. D.



$$-2k(x-y) = m\ddot{x}$$

$$\Rightarrow \quad m\ddot{x} + 2kx = 2ky$$

$$\Rightarrow \quad m\ddot{x} + k_{eq}x = k_{eq}y$$

$$k_{eq} = 2k = 2\,(3.519\times10^6 \text{ N/m}) = 7.308\times10^6 \text{ N/m}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{7.308\times10^6\text{ N/m}}{10^5\text{ kg}}} = 8.389 \text{ rad/s}$$

$$r = \frac{\omega_b}{\omega_n} = \frac{7.5\text{ rad/s}}{8.389\text{ rad/s}} = 0.8940$$

$$X = Y\frac{\sqrt{1+(2\zeta r)^2}}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = Y\frac{1}{|1-r^2|} = (0.1\text{ m})\frac{1}{|1-(0.8940)^2|}$$

$$= (0.1\text{ m})\,(4.98) = 0.498\text{ m}$$

[2.5절]

2.63 $m_0 = 12$ kg, $m = 100$ kg, $k = 3{,}000$ N/m, $N = 1{,}800$ rpm, $e = 100$ mm, $X_r = ?$

$k_{eq} = k + k = 2\ (3.0\times10^3\ \text{N/m}) = 6{,}000\ \text{N/m}$

$$\omega_r = \frac{(2\pi\ \text{rad})N}{60\ \text{s/min}} = \frac{(2\pi\ \text{rad})(1{,}800\ /\text{min})}{60\ \text{s/min}} = 188.5\ \text{rad/s}$$

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = \sqrt{\frac{6{,}000\ \text{kg/s}^2}{100\ \text{kg}}} = 7.746\ \text{rad/s}$$

$$r = \frac{188.5}{7.746} = 24.3$$

$$X_r = \frac{m_0 e}{m}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}} = (0.12)(0.10\ \text{m})\ \frac{(24.3)^2}{\sqrt{[1-(24.3)^2]^2}} = 0.01200\ \text{m}$$

$$= 12.00\ \text{mm}$$

2.67 $m = 150$ kg, $k = 1{,}000$ kN/m, $c = 600$ kg/s, $N = 3000$ rpm, $F_0 = 374$ N

$$\omega_n = \sqrt{\frac{1{,}000\times10^3\ \text{kg/s}^2}{150\ \text{kg}}} = 81.6\ \text{rad/s},\ \omega_r = \frac{(2\pi\ \text{rad})(3{,}000\ /\text{min})}{60\ \text{s/min}} = 314\ \text{rad/s}$$

$$r = \frac{314}{81.6} = 3.85, \qquad \zeta = \frac{600\ \text{kg/s}}{2(150\ \text{kg})(81.6\ \text{rad/s})} = 0.0245$$

(a) $$X = \frac{m_0 e}{m}\frac{\omega_r^2}{\sqrt{(\omega_n^2-\omega_r^2)^2+(2\zeta\omega_n\omega_r)^2}} \qquad (m_0\, e\, \omega_r^2 = F_0)$$

$$= \frac{F_0}{m\,\omega_r^2}\frac{r^2}{\sqrt{(1-r^2)^2+(2\zeta r)^2}}$$

$$= \frac{(374\ \text{N})}{(150\ \text{kg})(314\ \text{rad/s})^2}\ \frac{3.85^2}{\sqrt{(1-3.85^2)^2+[2(0.0245)(3.85)]^2}}$$

$$= 0.0271\times10^{-3}\ \text{m} = 0.0271\ \text{mm}$$

(b) $m_0/m = 0.01$

$m_0 = 0.01\ m = 0.01\ (150\ \text{kg}) = 1.50\ \text{kg}$

$$e = \frac{F_0}{m_0\,\omega_r^2} = \frac{374\ \text{N}}{(1.50\ \text{kg})(314\ \text{rad/s})^2} = 2.53\times10^{-3}\ \text{m} = 2.53\ \text{mm}$$ (교재 답 오류)